ESTIMATION OF SOIL COMPACTION PARAMETERS BASED ON THE ATTERBERG LIMITS

Abstract

This paper presents the relationship between the Atterberg limits and soil compaction parameters obtained by the correlation-regression analysis. The relations between the liquid limit \( w_l \), plastic limit \( w_p \), maximum dry density \( \rho_{max} \) and optimum moisture content \( w_{opt} \) are obtained on the basis of the results of laboratory tests measured on a large number of samples of clay core earthfill dams Rovni, Selova, Prvonek and Barje. The regression and correlation analysis were obtained empirical equations and diagrams. Based on the obtained, the value of the optimum moisture content and maximum dry density of knowing the Atterberg limits of plasticity can be estimated.

Keywords: compaction parameters, maximum dry density, optimum moisture content, Atterberg limits, liquid limit, plastic limit, regression analysis

1 INTRODUCTION

When making earthfill dams, the earth embankments and some other earthen structures in general, basically the problem is to define conditions of embedding material, and optimum conditions of compaction. Embedding, i.e. suitability of materials for building in and behavior of materials during compaction depends on geomechanical properties of materials. Some materials, such as sandy gravel is easier to compact, while in clay, especially clay of high plasticity, it is not [3]. Difficulties in the compaction of clayey material are closely related to the state of consistency index, where there is a dependency relationship of natural water content, plastic limit and liquid limits. This paper presents the relationship between the Atterberg limits \( (w_l, w_p) \), plasticity index \( (I_p) \) and soil compaction parameters \( (\rho_{max} \ and \ w_{opt}) \) obtained by the standard laboratory tests (SRPS U.B1.020:1980, SRPS U.B1.038:1997).

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These dependencies were the subject of many previous studies. The first correlation relationship between the liquid limit, plastic limit and the optimum moisture content was established Jumikis (1946). Ring et al. (1962), in addition to the Atterberg limits in correlation, included the average particle diameter and percentage of fine grained particles [16]. Al - Khafaji (1993) gave a correlation equation for maximum dry density and optimum moisture content, based on liquid limits and plastic limits, for four sites in Iraq, compared to the correlation equations obtained for the soil in the U.S. (Ingles and Metcalf, 1972). In addition, he has developed the correlation diagrams that have a practical application in the estimation of compaction parameters [1]. Pandian et al. (1997) proposed a series of predicted curves of soil compaction using the liquid limit and plastic limits. They also gave the special equations for so-called wet and dry parts of the curve compaction [12]. Blotz et al. (1998) also proposed two equations for estimation the compaction parameters of soil using the liquid limit for different compaction effort: standard and modified [2]. Sridharan and Nagaraj (2005) proposed two empirical equations for estimation the parameters of compaction using only plastic limits [19]. In contrast to them, Matteo (2009) and Noor (2011) gave the predictive models for estimation the compaction parameters of fine grained soils for the Standard Proctor test, based on the plastic limit, plasticity index and specific gravity of soils $\rho$ [10,11]. In recent years, in predicting the compaction parameters based on Atterberg limits, the method of artificial neural networks was used [8,9].

In this paper, the relations between the liquid limit $w_l$, the plastic limit $w_p$, maximum dry density $\rho_{\text{max}}$ and optimum moisture content $w_{\text{opt}}$ are obtained on the basis of the results of laboratory tests measured on representative samples of clay from the core earthfill dams Rovni, Selova, Prvonek and Barje [4,5,6,7]. The correlation and regression analysis was applied to define the statistical model using MS Excel (Analysis ToolPak), where the empirical equations and diagrams were obtained.

As indicators of relationship degree, i.e. correlation between the analyzed variables, the followings were used: $r$ - coefficient of simple linear correlation (Pearson coefficient), $R^2$ - coefficient of determination and $p$ - value which expresses the statistical significance of correlation i.e. level of significance [13]. Although simple linear regression models were obtained with a significant value to the strong coefficient of correlation ($0.73 < r < 0.85$) and the coefficient of determination $R^2$ ($0.53 < R^2 < 0.73$) and satisfying $p$-value ($p < 0.05$), the analysis is extended using the multiple linear regression, the introduction of calculation at the same time both $w_l$ and $w_p$.

2 RESULTS OF GEOMECHANICAL LABORATORY TESTING

Geotechnical laboratory testing of samples of clay were carried out during the geomechanical control of earthfill dams. Since a large number of samples, tested for statistical analysis, were selected, 72 representative samples were taken from the embedded core layer of earthfill dams in various stages of filling.
Figure 1 shows the average particle size distribution curve, and Figure 2 shows a diagram of plasticity the tested materials from the clay core of earthfill dams Selova, Prvonek, Barje and Rovni.

According to the generally accepted classification of soil (SRPS U.B1. 001: 1990), based on the identification-classification experiments: the grain-size distribution and Atterberg limits of tested material present:

- Low plasticity sandy clays embedded in the core earthfill dam Barje - CL.
- Medium to high plasticity silty clay, CL/CH, which originate from the core Prvonek earthfill dam,
- High plasticity silty clay CH from the core earthfill dam Rovni
- Clays embedded in the core earthfill dam Selova are sandy silty, medium to low plasticity CI-CL.

To determine maximum dry density and optimum moisture content of materials, the laboratory experiments were carried out by the standard Proctor compaction procedure (standard compaction energy E = 592 kJ/m³). The obtained results are shown in diagram in Figure 3.
Furthermore, the mathematical relationship between $\rho_{\text{dmax}}$ and $w_{\text{opt}}$ is presented in Figure 3.

3 STATISTICAL ANALYSIS AND DEFINING THE MODEL

Regression analysis is statistical tool that is used to define the analytical - mathematical models, i.e. functions between the independent (x) and dependent (y) variables. Strength of dependence between variables is determined using the correlation analysis [13]. Discussion and interpretations model is based on the coefficient of correlation $r$ and coefficient of determination $R^2$.

Value of coefficient of determination - $R^2$ indicates the representativeness of the model. The model is more representative of $R^2$ which is closer to one. The linear coefficient of correlation $r$ is a measure of correlation strength between variables x and y. According to Vukadinovic, if $r \leq 0.30$ there was no significant correlation, if $0.5 < r < 0.7$ correlation is significant, when $0.7 < R < 0.9$ correlation is strong, in the case where $r > 0.9$ very strong correlation [20]. So if the correlation coefficient was closer to one, the correlation was stronger.

The statistical significance of the model is defined using p - value or significance level p. If $p < 0.05$, $p < 0.01$ or $p < 0.1$ is acceptable to the model and the safety of $P > 95\%$, $P > 99\%$, or $P > 90\%$. In the case where $p > 0.05$ correlation was not significant, and then regardless of the value coefficient of correlation, a model should not be accepted and interpreted.

Although simple linear regression models, obtained (according to Vukadinovic, 1990) with the strong correlation coefficient values ($0.73 < r < 0.85$) and the coefficient of determination $R^2$ ($0.53 < R^2 < 0.73$) and satisfying p-value ($p < 0.05$), the analysis is extended using the multiple linear regression.

According to the method of Multiple Linear Regression – MLR, the evaluation of required variable is obtained on the basis of simultaneous use of a number of other independent variables. As for the parameters of soil density (maximum dry density $\rho_{\text{dmax}}$ and optimum moisture $w_{\text{opt}}$), six independent variables can be used in order to establish the most accurate according to their determination, as follows: E - energy compaction, G - percentage of gravel fractions, S - percentage of sand fraction, SF - percentage of...
fine fraction (clay and silt), \( w_l \) – liquid limit, \( w_p \) – limits of plasticity and \( I_p \) – plasticity index. According to Sivrakaya (2013), equation is as follows \[17,\]18\].

\[
[w_{\text{opt}}, \rho_{\text{dmax}}] = f(E, G, S, F, I_p, w_l, w_p)
\]  

(1)

The work has also introduced into equation both liquid limit \( (w_l) \) and plastic limits \( (w_p) \) simultaneously.

**The relationship between liquid limit and compaction parameters**

The first step analyzed the influence of \( w_l \) (independent variable) on the compaction parameters \( \rho_{\text{dmax}} \) and \( w_{\text{opt}} \) as dependent variables. Where the following equations were obtained:

\[
\rho_{\text{dmax}} = 2.088 - 0.008 w_l
r = 0.85, R^2 = 0.73, p<0.05
\]  

(2)

\[
w_{\text{opt}} = 0.239 w_l + 7.757
r = 0.83, R^2 = 0.69, p<0.05
\]  

(3)

Correlation between \( \rho_{\text{dmax}} \) and \( w_l \) is a negative linear correlation \( (\text{Fig. 4a}) \) with high coefficient of correlation \( r \) and satisfactory \( p \)-value \( (p<0.05) \). Correlation between \( w_{\text{opt}} \) and \( w_l \) is a positive linear correlation \( (\text{Figure 4b}) \) with slightly lower coefficient of correlation \( r \), and satisfactory \( p \)-value \( (p<0.05) \).

![Figure 4](image)

*Figure 4* Diagrams of relationship between the liquid limit and compaction parameters

**The relationship between the plastic limit and compaction parameters**

The second step analyzes the influence of \( w_p \) (independent variable) on the compaction parameters \( \rho_{\text{dmax}} \) and \( w_{\text{opt}} \) as dependent variables. The following equations were obtained:

\[
\rho_{\text{dmax}} = 2.229 - 0.023 w_p
r = 0.73, R^2 = 0.53, p<0.05
\]  

(4)

\[
w_{\text{opt}} = 0.742 w_p + 2.236
r = 0.78, R^2 = 0.61, p<0.05
\]  

(5)

Correlation between \( \rho_{\text{dmax}} \) and \( w_p \) is a negative linear correlation \( (\text{Vukadinovic, 1990}) \) with a strong coefficient of correlation \( r \) and satisfactory \( p \)-value \( (p<0.05) \). Correlation between \( w_{\text{opt}} \) and \( w_l \) is a positive linear correlation \( (\text{Fig. 5b}) \) with slightly higher coefficient of correlation, and satisfactory \( p \)-value \( (p<0.05) \).
The relationship between the plasticity index and compaction parameters

The third step analyzes the influence of plasticity index $I_p$ (independent variable) on parameters of density $\rho_{\text{max}}$ and $w_{\text{opt}}$ as dependent variables. As per definition, the plasticity index is the difference of liquid limit and plastic limit; it was interesting to analyze this effect. The following equations were obtained:

$$\rho_{\text{max}} = 1.948 - 0.0099 I_p$$

Figure 5 Diagrams of relationship between the plastic limit and compaction parameters

Correlation between $w_{\text{opt}}$ and $I_p$ is a positive linear correlation (Fig. 6a) with strong coefficient of correlation $r$, and satisfactory $p$-value ($p<0.05$).

$$w_{\text{opt}} = 0.276 I_p + 12.02$$

$ r = 0.78 $, $ R^2 = 0.65 $, $ p<0.05 $ \hfill (6)

$$w_{\text{opt}} = 0.276 I_p + 12.02$$

$ r = 0.73 $, $ R^2 = 0.57 $, $ p<0.05 $ \hfill (7)

The relationship between $\rho_{\text{max}}$ and $I_p$ is a negative linear correlation (Fig. 6a) with strong coefficient of correlation $r$ and satisfactory $p$-value ($p<0.05$).

In the fourth step, the analysis is extended using the method of multiple linear regression, introducing both $w_1$ and $w_p$ into equation.
\[ \rho_{\text{max}} = 2.14 - 0.007w_l - 0.005w_p \]
\[ r = 0.86, R^2 = 0.73, p < 0.05 \quad (8) \]
\[ w_{\text{opt}} = 4.18 + 0.16w_l + 0.323w_p \]
\[ r = 0.86, R^2 = 0.73, p < 0.05 \quad (9) \]

The relation obtained for \( \rho_{\text{max}} \) negative linear correlation with a strong coefficient of correlation \( r = 0.86 \) and satisfactory \( p \)-value (\( p < 0.05 \)).

Correlation obtained for \( w_{\text{opt}} \) is a positive linear correlation with strong coefficient of correlation \( r = 0.86 \), and satisfactory \( p \)-value (\( p < 0.05 \)).

It is seen analyzing the obtained equations that optimum moisture increases until maximum dry density decreases with increasing plastic properties of the soil. The equation obtained for optimum moisture content \( w_{\text{opt}} \) has stronger correlation with plastic limit \( w_p \), compared to the liquid limit \( w_l \) and plasticity index \( I_p \). However, the equations derived to calculate maximum dry density \( \rho_{\text{max}} \) show stronger correlation relationship (higher coefficient of correlation) with the liquid limit \( w_l \) and plasticity index \( I_p \).

Model extended the application of the Multiple Linear Regression, introducing into the equation both \( w_l \) and \( w_p \) giving the highest correlation coefficient \( r = 0.86 \) and the coefficient of determination \( R^2 = 0.73 \).

In addition to the linear regression, the logarithmic and exponential equations were also analyzed, but the best results with the highest coefficient of determination were given by linear equation. The linear correlation is far away from a practical point of view of the most acceptable.

CONCLUSION

Application of correlation - regression analysis was obtained by statistical models that define the relationship between the Atterberg limits: liquid limit \( w_l \), the plastic limit \( w_p \), plasticity index \( I_p \) and and soil compaction parameters: maximum dry density \( \rho_{\text{max}} \) and optimum moisture content \( w_{\text{opt}} \). Statistical analysis was carried out at 72 representative sample derived from the core of the earth dams Rovni, Selova, Prvonek and Barje. The test materials are of low to high plastic clay. For the shown statistical models, there was strong linear correlation with strong coefficient of correlation \( r \) and satisfactory \( p \)-value. As between the considered variables there are essential connections between (physical- mechanical properties of the soil), and analysis was carried out, the sufficient number of data were obtained with high degree of reliability on representative samples; the resulting models can be used to estimate the parameters compaction in the preliminary stages of the project, or preliminary assessment of suitability of any material from borrow pits controlling the quality of the earthfill structures.

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